THE INFLUENCE OF CIRCUIT PARAMETERS ON THE HARMONICS OF A PWM DC-TO-SINUSOIDAL INVERTER

Yuan Yu
TRW Systems Group
Redondo Beach, Calif.

Thomas G. Wilson
Duke University
Durham, N. C.

Ihsan M. H. Bábaá
Bell Telephone Labs.
Greensboro, N. C.

Sam Y. M. Feng
Duke University
Durham, N. C.

William A. Sander, III
Duke University
Durham, N. C.

Summary

Dc-to-sinusoidal inverters are extensively employed in power-conditioning systems in aerospace applications. With the availability of high-voltage fast-switching power semiconductors, it has become practical to utilize the pulse-width-modulation (PWM) technique to generate a voltage waveform consisting of a repeating pattern of high-frequency pulses which is subsequently filtered to obtain a low-frequency sinusoidal output. These pulses have the same amplitude, but their widths are modulated to vary in accordance with a sinusoidal reference. This paper is concerned with a harmonic analysis of the PWM voltage waveform. The dependence of the amplitudes of the odd harmonics up to the nineteenth on the direct input voltage, on the sinusoidal reference amplitude, on the sinusoidal reference frequency, and on the switching frequency of the dc-to-sinusoidal inverter is given. This information reveals that choosing a filter with a cutoff frequency slightly below the switching frequency of the inverter may not sufficiently attenuate all of the significant harmonics. The paper supplies data which enable the designer to choose parameters such as the switching frequency, the amplitude of the sinusoidal reference, and the output filter so that the efficiency and the size and weight of the inverter may be optimized.

Introduction

The dc-to-sinusoidal inverters commonly utilized in aerospace power-conditioning systems operate at switching frequencies that are very high in relation to the desired sinusoidal output frequency which is typically 400 Hertz. This approach commonly can be accomplished by applying pulse-width-modulation (PWM) techniques which generate a repeating pattern of high-frequency square pulses as the unfiltered output waveform. Each pulse has the same amplitude which is equal to the dc input voltage to the inverter. This high-frequency approach offers the potential of enabling a considerable reduction in the size and weight of output-filter components. Many methods applying PWM techniques have been used to generate the unfiltered output waveforms.[1,2,3,4] The objective in most cases is to program the widths of the pulses in the pattern so that the amplitudes of the lower order harmonics of the sinusoidal output are kept small.

This work was sponsored by the National Aeronautics and Space Administration under Grant No. NGL 34-001-001 to Duke University.

An illustrative PWM waveform with an unusually small number of crossover angles and its fundamental component shown by the dotted curve are given in Fig. 1(a). The waveform shown is an odd

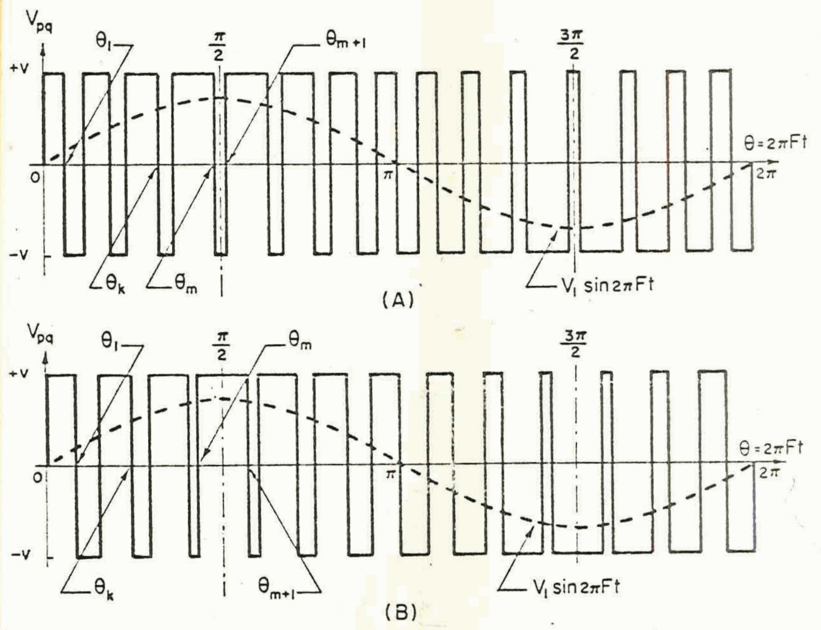


Figure 1(a). PWM waveform for odd m.
(b). PWM waveform for even m.

function with quarter-wave symmetry and thus can be expressed as a series of sine waves containing only odd harmonics of the fundamental. Fourier analysis of this waveform shows that the peak amplitude V_n of the nth odd harmonic is

$$V_n = (4V/n\pi)\left[1 - 2\cos n\theta_1 + 2\cos n\theta_2 - \dots - 2\cos n\theta_m + \cos(n\pi/2)\right] \quad (1)$$

where n = 1,3,5,... Since (1) contains m variables θ_1 to θ_m, it is theoretically possible by properly controlling each pulse width to eliminate m different harmonics, e.g., to reduce V_3, V_5,..., V_{2m+1} to zero. The principle of this technique has been utilized to eliminate the third and the fifth harmonics.[1] However, if this technique is extended to eliminate additional higher-order harmonics, the sophistication needed to control the successive pulses and the physical complexity of the control circuits increase progressively.

It is desirable to employ a relatively simple switching control and at the same time achieve a size and weight reduction through high-frequency operation. One way this can be accomplished is by using a small, externally-generated sinusoidal reference of the desired fundamental frequency F to modulate the widths of successive pulses to produce a waveform similar to Fig. 1(a).[2,3,4] For a zero amplitude of sinusoidal reference corresponding to no pulse-width modulation of the output, the output-voltage waveform consists of identical symmetric square pulses of constant frequency f. For a nonzero sinusoidal-reference amplitude, the widths of successive pulses are controlled by various techniques to vary in accordance with the given reference. In this type of configuration, the angles θ_1 to θ_m are no longer adjusted individually to eliminate selected harmonics. Instead, for a certain amplitude of the sinusoidal reference, there is a unique set of crossover angles $\theta_1, \theta_2, \theta_3, \ldots \theta_m$. Without individual control of each angle, the complexity of the control circuit is reduced. On the other hand, since the angles are not specifically chosen to eliminate any particular V_n, these harmonics can no longer be expected to vanish. A low-pass filter with a cut-off frequency that is significantly higher than the fundamental reference frequency is used in the output to attenuate to negligible levels the harmonics produced by this method.

It becomes evident later, when the data from the computer-aided analysis is presented, that it is possible when employing PWM techniques to keep the magnitude of the lower order harmonics of the fundamental frequency small simply by choosing a switching frequency that is very high. However, choosing an extremely high switching frequency does not solve all of the problems of the designer. Higher switching frequencies increase the losses; consequently, in this type of dc to sinusoidal inversion, efficiency and harmonic content are trade-offs. To achieve the best overall performance of an inverter with a given set of specifications, a compromise must be made. This compromise requires information concerning the variation of the amplitudes of the harmonics with the circuit parameters. The fundamental component of the output waveform is known to increase rather linearly with the amplitude of the sinusoidal reference.[3] Information concerning the variations of the amplitudes of the harmonics with the amplitude of the sinusoidal reference and with the switching frequency f has not been available and is presented in this paper.

Since rather stringent limitations are normally imposed on the allowable harmonic distortion of the ac output voltage, the prediction of the amplitude of each of the low-frequency harmonics is very important in designing dc-to-sinusoidal inverters using pulse-width-modulation techniques.

One objective of this paper is to perform a harmonic analysis of sine-wave-pulse-width-modulated waveforms of the type shown in Fig. 1. The analysis focuses on calculating the amplitudes $V_1, V_3, V_5, \ldots V_{19}$ of the fundamental and the odd harmonics, as these normally are the ones of greatest practical concern. These amplitudes are related to the fundamental frequency F, to the switching frequency f of the symmetric pulses had the modulation been zero, and to a quantity defined later as the amplitude ratio M/E. With the aid of a digital computer, the ratios V_n/V_1 for different M/E as functions of f/F have been calculated and are plotted in the form of curves. Also determined and plotted as a function of f/F are the natural frequencies of L-section filters needed to attenuate any remaining harmonics to certain specified percentages of the fundamental amplitudes. These results can be employed to facilitate the selection of a switching frequency, f and an amplitude ratio M/E for an inverter having certain specifications on its fundamental output frequency, amplitude ratio, efficiency, and its harmonic content.

In the following presentation, the circuit used to generate output waveforms similar to Fig. 1 is described first. Computer analysis of the harmonics is then carried out to establish the harmonic amplitudes as functions of the amplitude ratio M/E and the frequency ratio f/F. These results are then used to determine the low-pass-filter natural frequencies F_o as functions of f/F. These functions are expressed explicitly in the form of curves so that they can be employed readily for design purposes. From these results of harmonic content, guidelines are presented which aid the designer in selecting the circuit parameters.

Inverter Circuit

The inverter circuit shown in Fig. 2 is used to generate a sine-wave-modulated output waveform similar in form to those shown in Fig. 1. The circuit has been described in Ref. 3, and therefore is reviewed here only briefly. It provides an open-loop-regulated low-frequency sine wave without the need of either a power transformer or a fundamental-frequency filter. The inverter is composed of a High-Frequency Pulse-Width Modulator with four output windings N5 to N8 which supply base drives to four power transistors Q3 to Q6 in the Bridge Chopper. Direct voltage E in conjunction with a sinusoidal reference $M \sin 2\pi Ft$ is used to supply each half of the pulse-width modulator. If M is zero, the modulator would be a symmetrical multivibrator; the frequency of its symmetric output pulses would be

$$f = E/4N\phi_s \qquad (2)$$

where N is the number of turns for winding N1 as well as N2, and ϕ_s is the saturation flux of square-loop core T1. The sinusoidal reference with amplitude M<E and frequency F which is

normally much less than f is used for the modulation purpose. When Q2 conducts during the time interval $\theta_{k-1}/2\pi F$ to $\theta_k/2\pi F$, which corresponds to

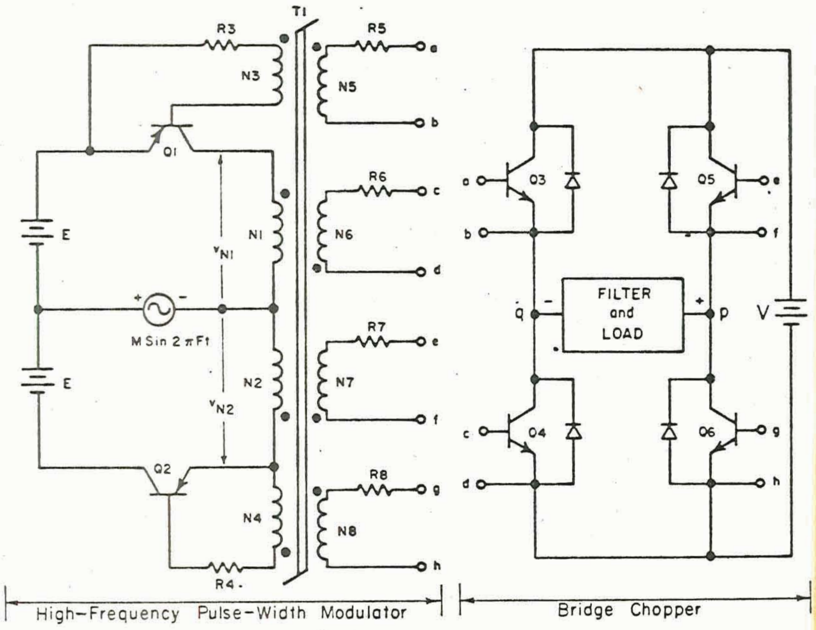


Figure 2. Schematic diagram of a sine wave PWM inverter.

angles θ_{k-1} and θ_k in Fig. 1, voltage v_{N2} across winding N2 is M(sin $2\pi Ft$) - E. When Q1 conducts subsequently between $\theta_k/2\pi F$ and $\theta_{k+1}/2\pi F$, voltage v_{N1} across N1 is M(sin $2\pi Ft$) + E. Analytically,

$$\int_{\theta_{k-1}}^{\theta_k/2\pi F} (E - M \sin 2\pi Ft)dt = 2N\phi_s \qquad (3)$$

and

$$\int_{\theta_k/2\pi F}^{\theta_{k+1}/2\pi F} (E + M \sin 2\pi Ft)dt = 2N\phi_s \qquad (4)$$

Since normally F<<f, many high-frequency pulses are thus generated in each of the windings N5 to N8 within a fundamental period 1/F of the sinusoidal reference. The amplitudes of these voltage pulses form sinusoidal envelopes proportional to M(sin $2\pi Ft$) ± E on N_1 and N_2 and the widths of successive pulses vary inversely with the amplitudes of the envelopes. These pulses are then used to control Q3 to Q6 of the bridge chopper.

As can be seen from the winding polarities shown in Fig. 2, these control pulses cause Q3 and Q6 to conduct with Q2. With input voltage V across two of the bridge terminals, the unfiltered output voltage v_{pq} across the other two terminals p and q has the pattern of alternating pulses with amplitude ±V as shown in Fig. 1. The fundamental frequency of v_{pq} is F. A small low-pass filter is inserted between terminals p and q and the load to provide the desired fundamental sinusoidal output voltage.

Harmonic Analysis

In order to perform a Fourier series analysis of the output voltage wave v_{pq} and determine its harmonic content, it is necessary that the wave be periodic. In the general case, the PWM modulated wave v_{pq} may have a period which is not an integral multiple of the period of the modulating voltage M sin $2\pi Ft$. Such a case presents no theoretical difficulty but can lengthen considerably the time required to carry out practical calculations. For this reason and since ample design data can be obtained, only two special cases are studied. Both require that the modulated wave possess quarter-wave symmetry. Figure 1 illustrates these two cases: Fig. 1(a) shows v_{pq} having an odd number m of crossover angles in $\pi/2$ and Fig. 1(b) shows the situation when v_{pq} has an even number m of crossover angles in $\pi/2$. In both cases, the first step is to calculate values of the crossover angles $\theta_1, \theta_2, \theta_3, \ldots \theta_{m+1}$ for a given set of E, M, F, and volt-second capacity $D = 2N\phi_s$ of core T1. In general, $\pi/2$ may occur anywhere between θ_m and θ_{m+1}. The desired quarter wave symmetry requires that $(\theta_m + \theta_{m+1})/2 = \pi/2$ which is obtained mathematically by adjusting slightly, using linear extrapolation, the numerical value of the parameter D. A set of crossover angles are computed for each new value of D until $(\theta_m + \theta_{m+1})/2 = \pi/2$ and the resulting modulated wave is then analyzed and put into the Fourier series form

$$V_{pq} = \sum_{n=1}^{\infty} V_n \sin n\omega t \qquad (5)$$

where n = 1,3,5,...

A single Fourier analysis can be performed for both cases, m odd and m even since the only difference in the Fourier coefficients for the two cases is in the number of terms in their series. The analysis then yields

$$V_n = \frac{4V}{n\pi}[1 - 2\cos n\theta_1 + 2\cos n\theta_2 - \ldots + (-1)^{m+1} 2\cos n\theta_{m-1} + (-1)^m 2\cos n\theta_m + (-1)^{m+1} \cos n\,\pi/2] \qquad (6)$$

as a general expression for the peak voltage V_n of the nth harmonic of the PWM waveform.

Results

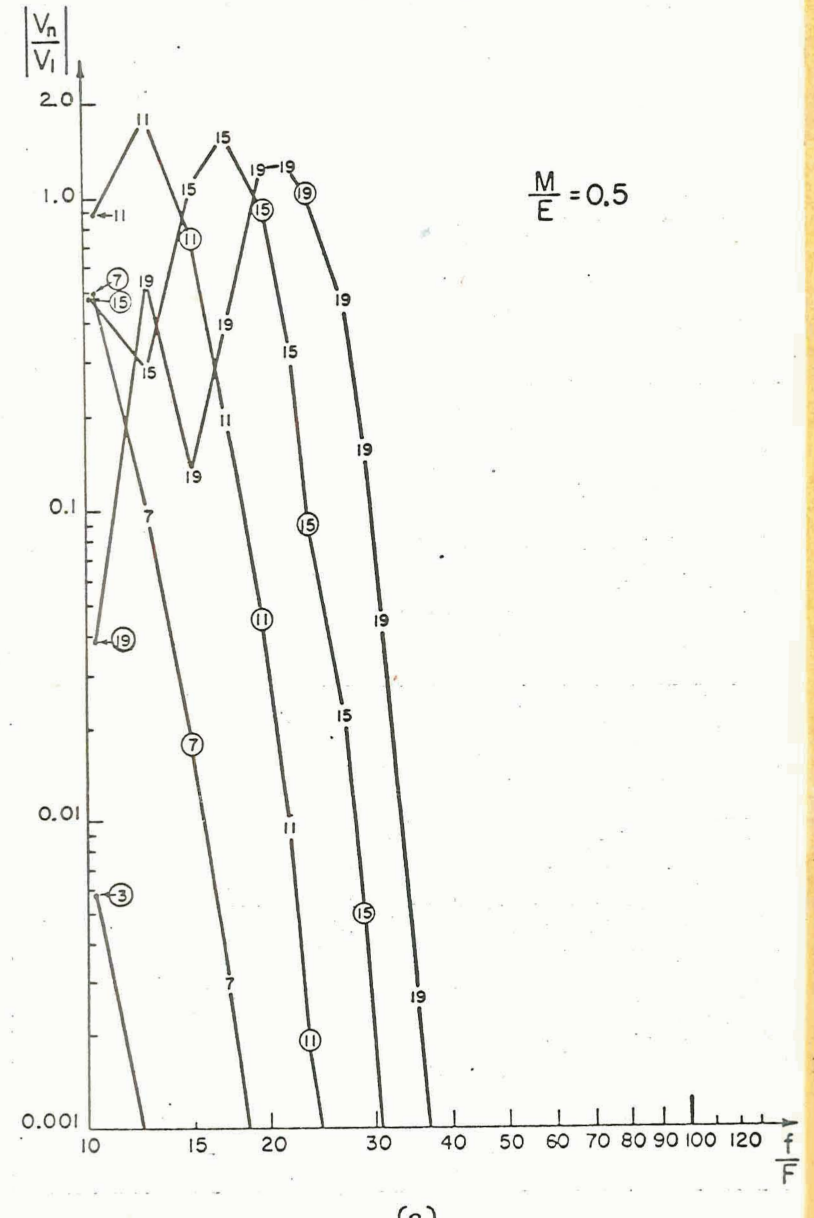
The amplitudes of the fundamental and its odd harmonics up to the 19th were calculated by digital computer according to (6) for both cases shown in Fig. 1, i.e., when m is odd and when m is even. For each value of f/F the condition that $(\theta_m + \theta_{m+1})/2 = \pi/2$ was satisfied by adjusting D. These calculations were made with E = 10 volts and M = 5, 7, and 9 volts. The resulting data are shown plotted for clarity in two figures for each value of M/E: in Figs. 3(a) and 3(b) for M/E = 0.5, in Figs. 4(a) and 4(b) for M/E = 0.7, and in Figs. 5(a) and 5(b) for M/E = 0.9.

A curve for the fundamental voltage V_1 in each case is not plotted because the data indicate that V_1 is independent of f/F and is linearly related to both M/E and V by the relation[3]

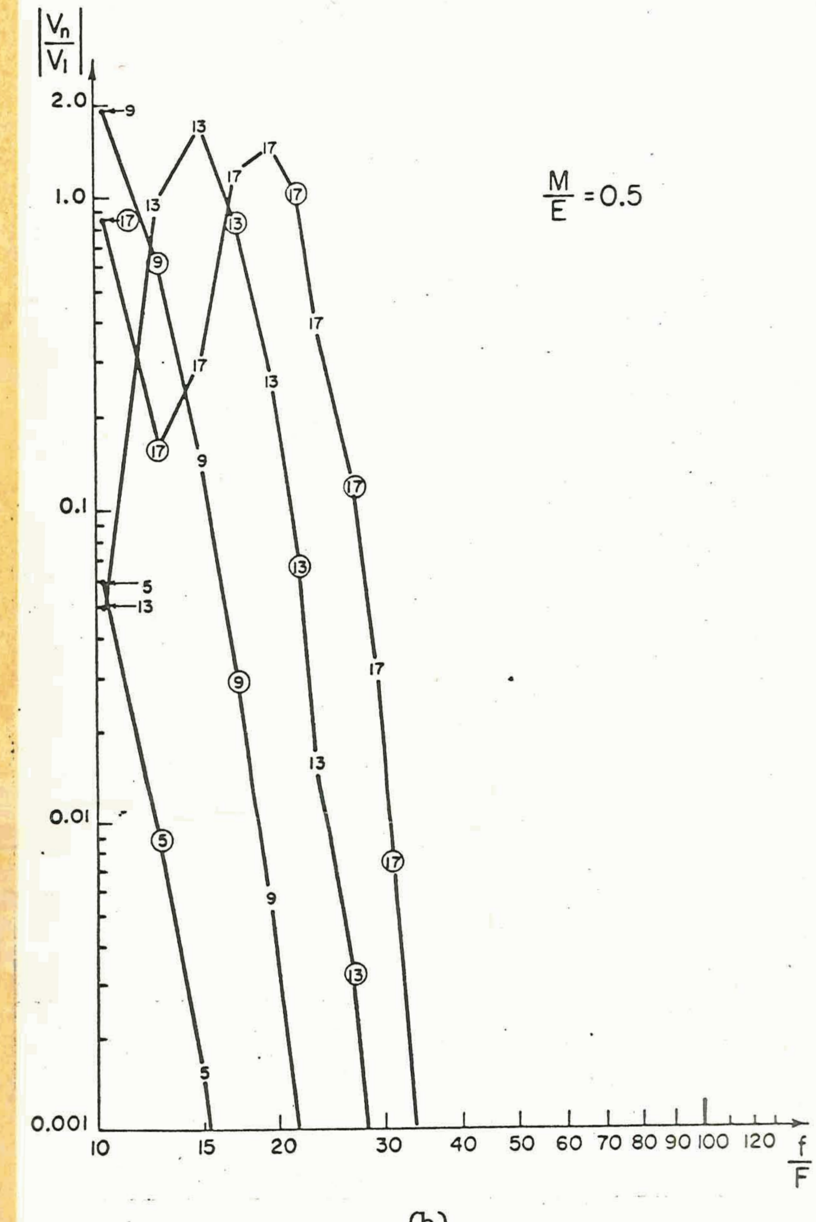
$$V_1 \cong V \cdot \frac{M}{E} \tag{7}$$

The actual data points are indicated by the number of the harmonic which they represent.

In interpreting the data of the figures it should be particularly noted that the plotted points are connected by straight line segments rather than smooth curves to emphasize the fact that computer results were obtained only at certain discrete f/F ratios. Although it is evident from the figures that the absolute-value amplitudes $|V_n/V_1|$ of many of the harmonics fluctuate significantly especially for small values of f/F, the computer solutions show that the V_n of certain harmonics are negative and that others actually change sign passing through zero at certain values of f/F. It is this property that allows the reduction of a particular harmonic to zero amplitude by proper selection of the crossover angles θ_k as discussed in Ref. 1. The data points corresponding to negative values of V_n are shown with circles surrounding the harmonic number. A more exact portrayal would show the $|V_n/V_1|$ curve going to zero amplitude at some f/F value each time there is a change in sign. But since this critically-tuned property is not being used, data points have simply been connected by straight line



(a)



(b)

Figure 3. Harmonic content as a function of f/F with M/E = 0.5.

segments to emphasize only the major trends.

It can be concluded from an examination of the three figures that every harmonic from the third through the nineteenth for each value of M/E has an appreciable amplitude for small values of f/F but that as f/F is increased the lower-order harmonics one by one in turn decrease sharply. These curves, therefore, support the earlier statement that the harmonic content can be kept extremely small for a given amplitude ratio M/E and fundamental frequency F by choosing a very large switching frequency f.

It can also be seen by comparing corresponding figures for different values of M/E that the corresponding harmonic amplitudes at given values of f/F normally become much greater as the amplitude ratio M/E is increased. The harmonic content may be kept small by minimizing M/E; but, at the same time, the input voltage V must be increased according to (7) if the output voltage is to remain the same. Consequently, in order to obtain a desired V_1 with smaller harmonic distortion, it is advantageous to use a relatively high input voltage and a small amplitude ratio. Fortunately, the selection of a close-to-unity amplitude ratio M/E, such as 0.9, is becoming less necessary since high-voltage fast-switching power transistors with voltage ratings to 400 volts are becoming readily available.

In addition to having minima, the amplitudes of certain harmonics are seen to have maximum values which can be greater even than the amplitude of the fundamental. Because of this, the designer must exercise a certain amount of caution in selecting the cutoff frequency for the output filter. In order to aid the design, computer calculations were made to determine the normalized natural frequencies $F_o/F = 1/(2\pi F\sqrt{LC})$ of each simple ideal L-section filter that would attenuate each harmonic amplitude to within a specified percent at a specified frequency ratio f/F. From this set of natural frequencies, the lowest was chosen so as to guarantee that all of the harmonics would be attenuated to within the specified percent. These calculations were performed for the same frequency ratios at which the harmonic amplitudes had been calculated and for percentages of 1, 2, 3, 5, 7, and 10. The attenuation characteristic of an unloaded filter assuming ideal inductive and reactive components, i.e., zero damping, was assumed in order to assure sufficient attenuation, since

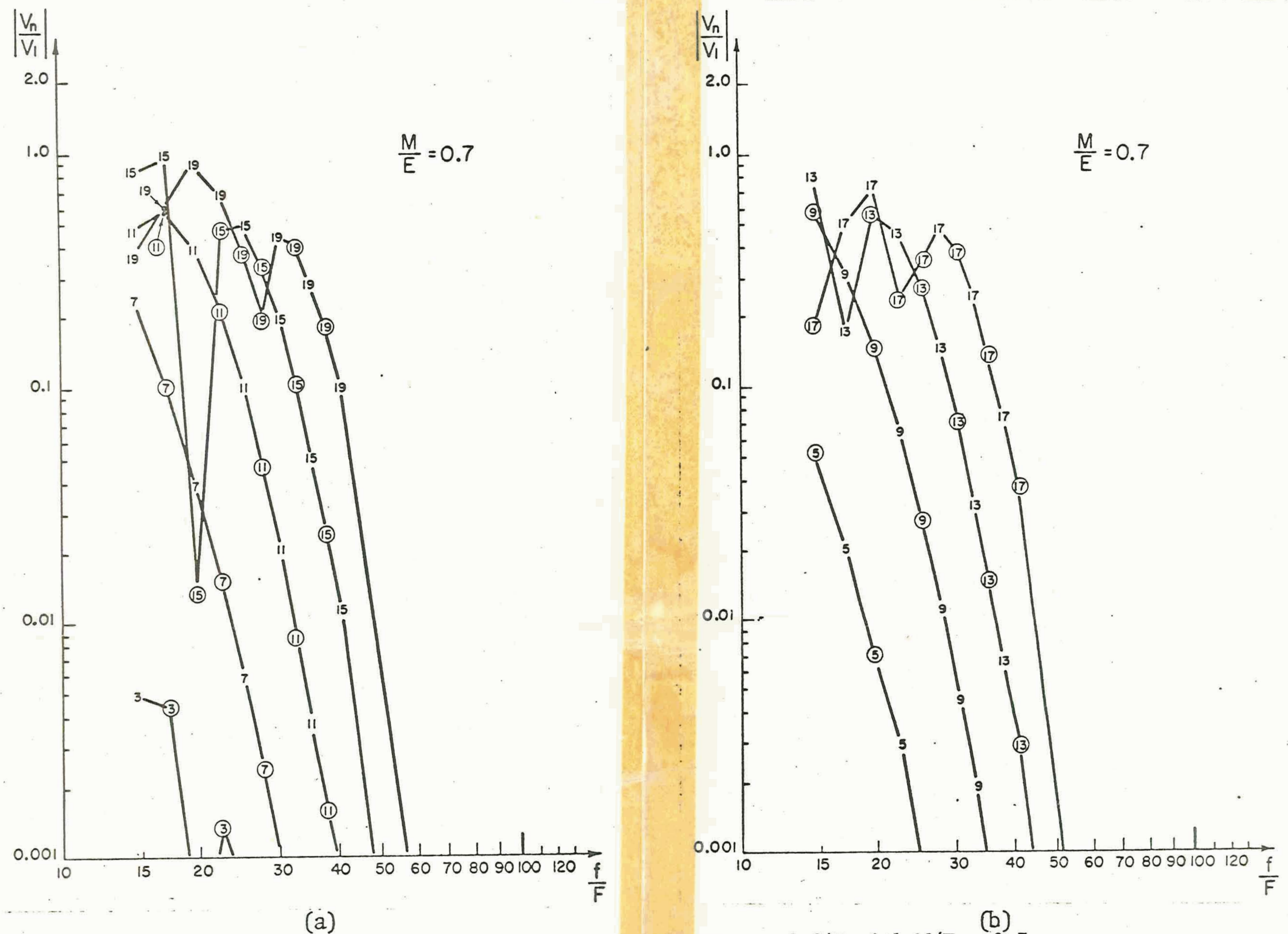


Figure 4. Harmonic content as a function of f/F with M/E = 0.7.

any damping caused by nonideal components and the load increases the attentuation.[5] The resulting normalized natural frequencies F_o/F are plotted in Figs. 6, 7, and 8 as functions of f/F. The numbers on the curves indicate the actual calculated data points and the order of the harmonic whose amplitude determines the required natural frequency for the filter. An L-section filter with an F_o selected from these curves in most cases guarantees that each odd harmonic up to the 19th be less than the chosen percentage at the chosen f/F. It is possible however, that a harmonic of frequency near F_o may have, between terminals p and q, an amplitude considerably lower than the percentage required by the output specification; but this same harmonic, due to the resonance characteristic of the filter with very small damping, may be larger at the load terminals than the amplitude of the harmonic which dictated the selection of F_o. Usually difficulty of this sort can be circumvented by using a larger value of L and a smaller value of C still for the same F_o.

It is worthwhile to point out that for M/E = 0.5 and 0.7 in Figs. 6 and 7, there is a frequency ratio above which the normalized natural frequency F_o/F increases very rapidly. A frequency ratio slightly greater than this value is beneficial from the viewpoint of reducing the size and weight of the filter; however, the inverter switching losses may become intolerable limiting the operation to a smaller f/F. The curves for M/E = 0.9 in Fig. 8 show a much less pronounced rise in F_o/F and the curves are shifted to higher values of f/F which indicates again that harmonic amplitudes are much greater near unity M/E for specified values of f/F. It can also be seen that the order of the harmonic which determines the filter natural frequency increases with f/F. Figures 6, 7, and 8 can be of particular value to the designer when a small frequency ratio f/F or a large amplitude ratio M/E must be employed. For example, if it is necessary that M/E = 0.7, f/F = 20, and the harmonic amplitude of each harmonic up to the 19th be less than 10 percent, then the designer can use Fig. 7 to select a filter with F_o = 5F.

Experimental data on harmonic content and some examples of the output filter design were taken to verify the computer results. Close agreement between experimental and computed results was obtained consistently. One example

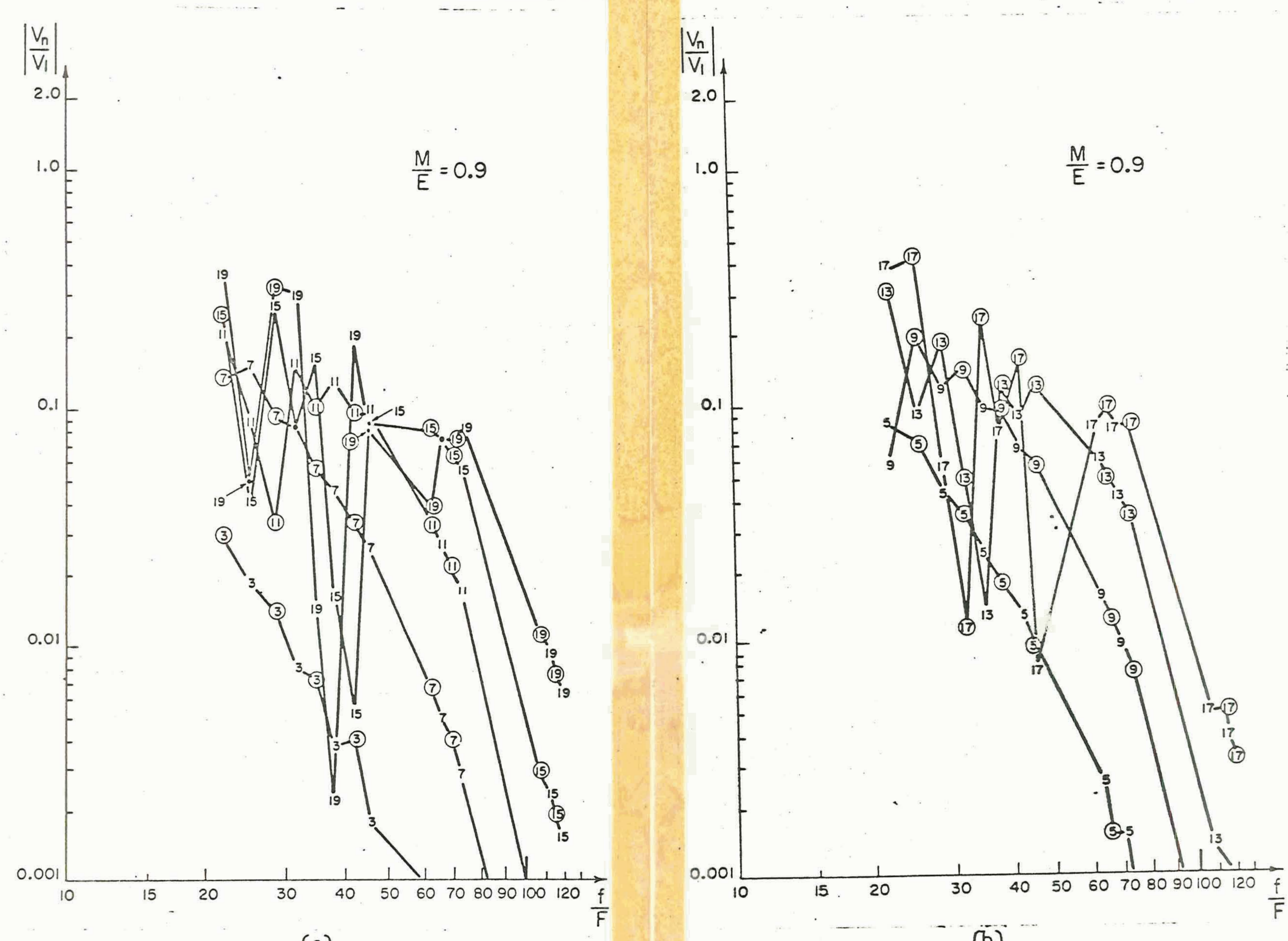


Figure 5. Harmonic content as a function of f/F with M/E = 0.9.

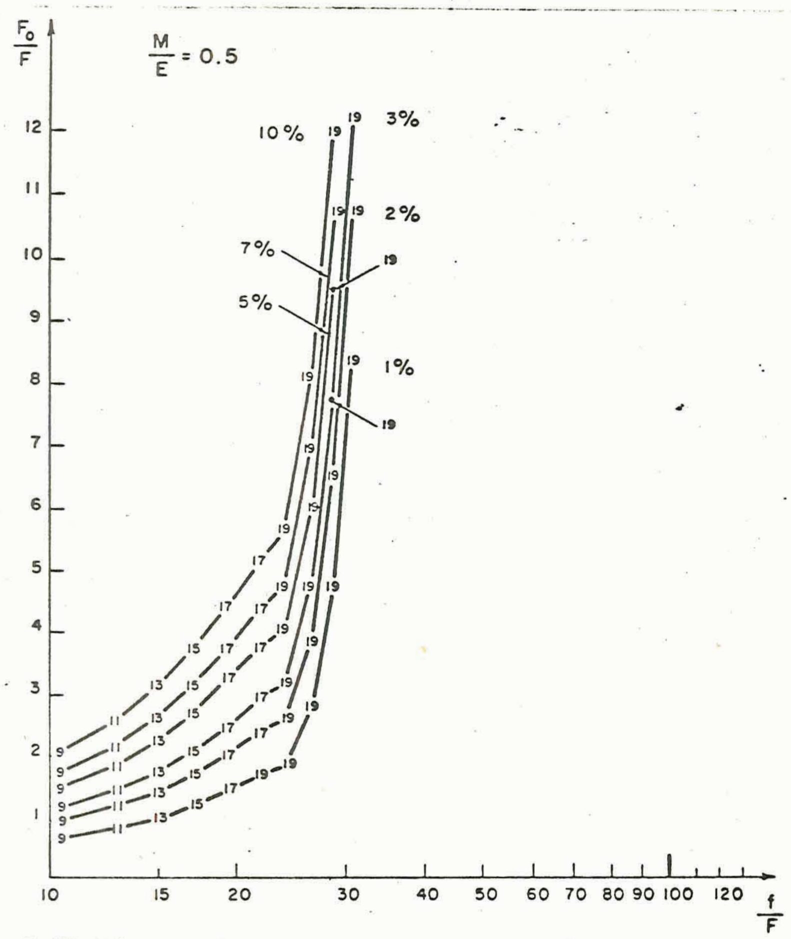


Figure 6. Filter design curves for M/E = 0.5.

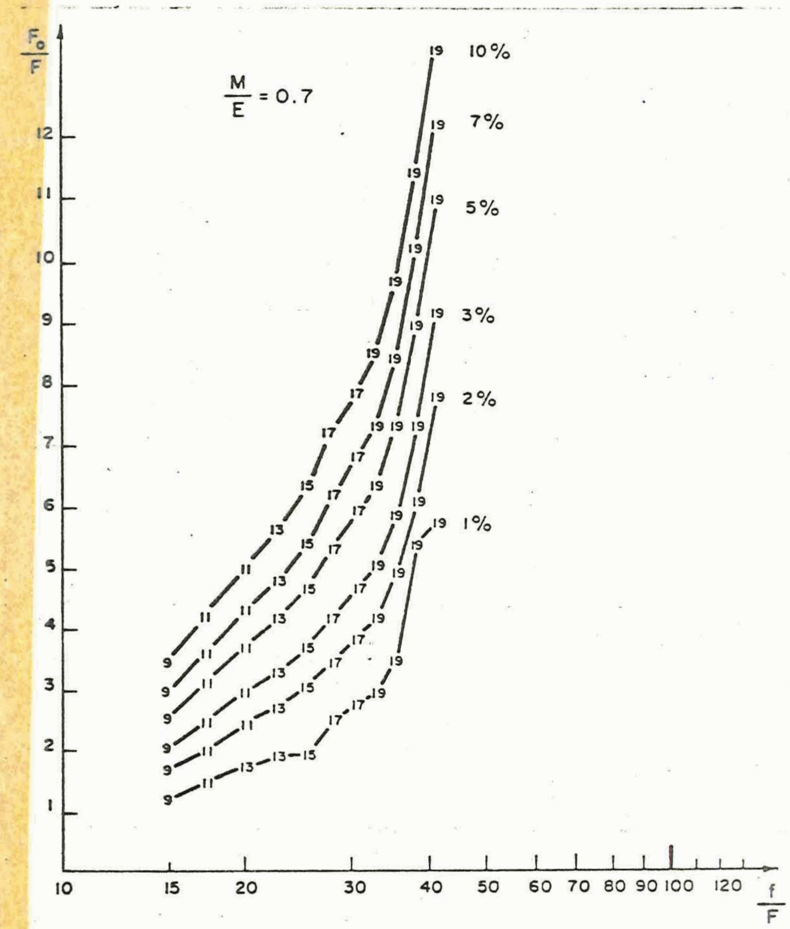


Figure 7. Filter design curves for M/E = 0.7.

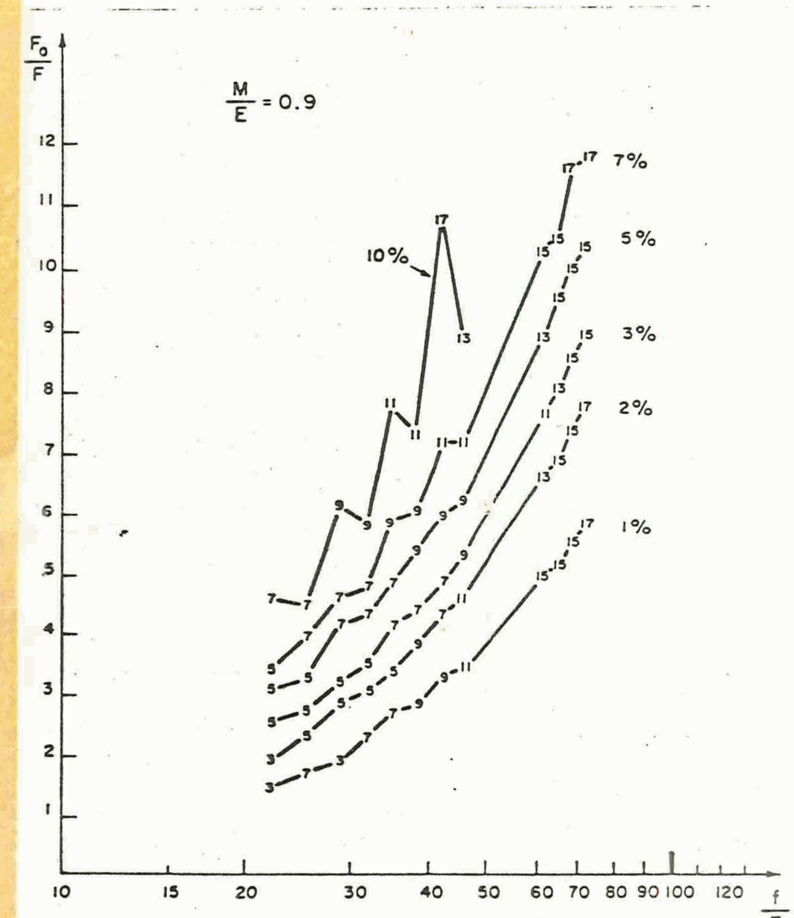


Figure 8. Filter design curves for M/E = 0.9.

illustrating the experimental data taken is given in Figs. 9 and 10 in the form of spectrum bar graphs for M/E = 0.5 and f/F = 21.72. Fig. 9 shows the harmonic content of the unfiltered sine-wave-modulated PWM wave V_{pq} as measured with a spectrum analyzer, and Fig. 10 shows the measured harmonic content after the insertion of an L-section filter for the case of $F_o/F = 3.69$. These results compare favorably with those predicted by Fig. 3 for the individual harmonic amplitudes and with Fig. 6 for the order of the maximum-amplitude harmonic and the percentage of the harmonic at the load. The experimental results also show that the amplitude of the third harmonic at the output of the filter is four percent whereas its amplitude at the input of the filter is not detectable. This illustrates what was discussed earlier that a lower-order harmonic in the vicinity of the natural frequency of the filter may actually be amplified. Therefore, consideration of this effect should influence the selection of the L and C to obtain the necessary F_o.

Conclusion

In this paper, a computer-aided analysis has been made on the harmonic content of the output of a dc-to-sinusoidal inverter employing a pulse-width-modulated waveform. Results obtained indicate that the fundamental component changes rather linearly with the amplitude ratio M/E and the input voltage V and is independent of the switching-frequency-to-fundamental-frequency ratio f/F. In contrast to the fundamental amplitude, the amplitudes of the odd harmonics increase very significantly with the amplitude ratio M/E. Consequently

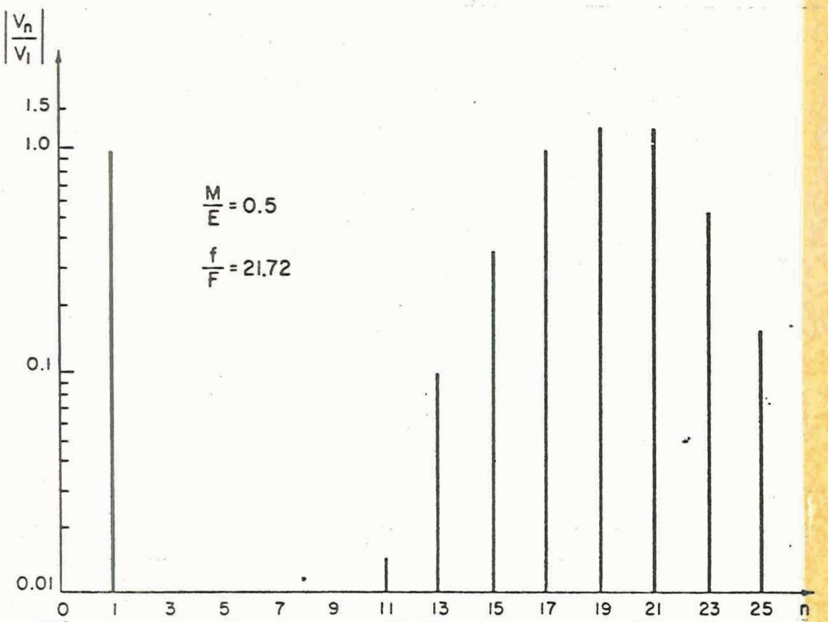


Figure 9. Spectrum of the unfiltered PWM waveform V_{pq}.

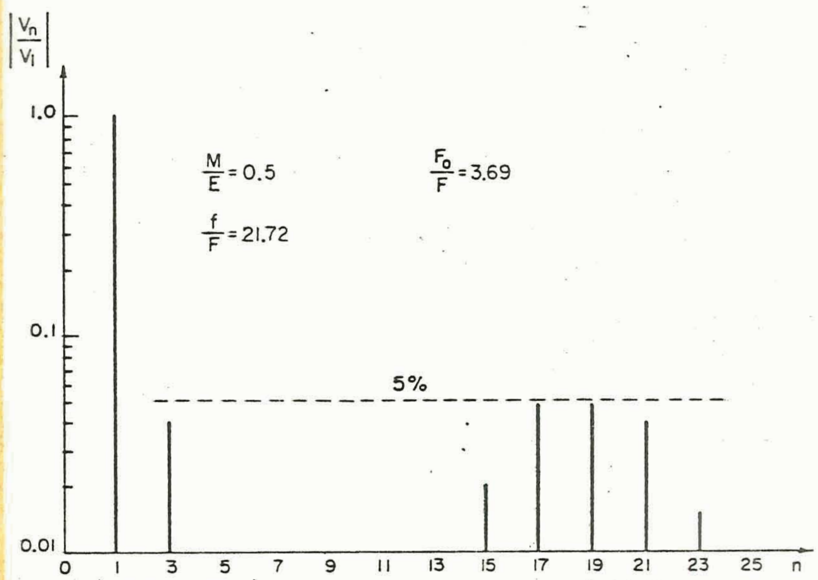


Figure 10. Spectrum of the filtered PWM waveform using L = 5 mH and C = 5 μF.

from the viewpoint of reducing the harmonic content, it is quite undesirable to use an amplitude ratio that is close to unity. It was found that the harmonic amplitude variation with the frequency ratio f/F was highly nonlinear with peaks and valleys. Therefore, simply choosing a filter cutoff frequency slightly below the switching frequency could easily be insufficient to attenuate all the significant harmonics. For these reasons, the curves presented in this paper which give the harmonic amplitudes and the natural frequencies of the L-section filters which attenuate the harmonics to fixed percentages of the fundamental amplitude are very useful in providing information to be used in optimizing the design of sine-wave PWM inverters.

References

1. R. E. Morgan, "Bridge-Chopper Inverter for 400 cps Sine Wave Power," IEEE Trans. Aerospace, Vol. AS-2, pp. 993-997, April 1964.

2. W. V. Peterson and R. J. Resch, "5 KW Pulse Width Modulated Static Inverter," TRW Equipment Laboratories, NASA Rept. CR-54872, December 1965.

3. Y. Yu, T. G. Wilson, I. M. H. Bábaá, S. Y. M. Feng, and E. T. Moore, "Static DC to Sinusoidal AC Inverter Using Techniques of High-Frequency Pulse-Width Modulation," IEEE Trans. Magnetics, Vol. MAG-3, No. 3, pp. 250-256, September 1967.

4. R. J. Ravas, P. F. Pittman, and R. W. Briggs, "Staggered Phase Carrier Cancellation - A New Circuit Concept for Lightweight Static Inverters," Supplement to IEEE Trans. Aerospace and Electronic Systems, Vol. AES-3, No. 6, pp. 432-444, November 1967.

5. John J. D'Azzo and Constantine H. Houpis, Feedback Control System Analysis and Synthesis, McGraw-Hill Book Company, Inc., New York, N.Y., pp. 184-187, 1960.